

# An Abstract Machine for Concurrent Haskell with Futures

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# Motivation

- **Concurrent Haskell** (Peyton Jones, Gordon, Finne 1996) extends Haskell by concurrency
- The **process calculus CHF** (S., Schmidt-Schauß 2011) models **Concurrent Haskell with Futures** operational semantics inspired by (Peyton Jones, 2001)
- Futures allow a **declarative programming style** for concurrency
  - **Future** = Variable whose value becomes available in the future
  - Our futures are **concurrent, imperative, implicit**
  - **implicit synchronisation by data dependency**

do

`x1 <- future e1`

`x2 <- future e2`

`some actions`

`print(x1+x2)`



`x1 <= e1`

`| x2 <= e2`

`| -  $\xleftarrow{\text{main}}$  do some actions`

`print (x1+x2)`

# Issues

Operational semantics of CHF given in (S.,Schmidt-Schauß 2011)

- Appropriate for **mathematical reasoning** on **contextual equivalence** w.r.t. may- and should-convergence
- **but its definition is complex**
- **not** obvious how to implement

## In this work:

- Design an **abstract machine** for CHF based on the machines of (Sestoft 1997) for lazy evaluation
- which can be implemented **easily** (prototype exists)
- show **correctness** of the abstract machine w.r.t. may- and should-convergence

# The Process Calculus CHF: Syntax

## Processes

$$P, P_i \in Proc ::= P_1 \mid P_2 \mid \nu x.P \mid x \leftarrow e \mid x = e \mid x \mathbf{m} e \mid x \mathbf{m} -$$

A process has a **main thread**:  $x \xleftarrow{\text{main}} e \mid P$

## Expressions & Monadic Expressions

$$e, e_i \in Expr ::= me \mid x \mid \lambda x.e \mid (e_1 e_2) \mid \text{seq } e_1 e_2 \mid c e_1 \dots e_{\text{ar}(c)}$$

$$\mid \text{case}_T e \text{ of } \dots (c_{T,i} x_1 \dots x_{\text{ar}(c_{T,i})} \rightarrow e_i) \dots$$

$$\mid \text{letrec } x_1 = e_1 \dots x_n = e_n \text{ in } e$$

$$me \in MExpr ::= \text{return } e \mid e_1 \gg= e_2 \mid \text{future } e$$

$$\mid \text{takeMVar } e \mid \text{newMVar } e \mid \text{putMVar } e_1 e_2$$

## Types

$$\tau, \tau_i \in Typ ::= (T \tau_1 \dots \tau_n) \mid \tau_1 \rightarrow \tau_2 \mid \text{IO } \tau \mid \text{MVar } \tau$$

Standard monomorphic type system

# Operational Semantics

## Operational Semantics: Reduction $P_1 \xrightarrow{CHF} P_2$

- Small-step reduction  $\xrightarrow{CHF}$
- Rules are closed w.r.t. structural congruence and process contexts
- Reduction rules for **monadic computation** and **functional evaluation**

Some rules:

**(fork)**  $x \leftarrow \mathbb{M}[\text{future } e] \xrightarrow{CHF} \nu y. (x \leftarrow \mathbb{M}[\text{return } y] \mid y \leftarrow e)$ ,  $y$  fresh

**(lbeta)**  $\mathbb{L}[(\lambda x. e_1) e_2] \xrightarrow{CHF} \nu x. (\mathbb{L}[e_1] \mid x = e_2)$

$\mathbb{L}$ -contexts:  $\mathbb{L} ::= x \leftarrow \mathbb{M}[\mathbb{F}]$

$\mid x \leftarrow \mathbb{M}[\mathbb{F}[x_n]] \mid x_n = \mathbb{E}_n[x_{n-1}] \mid \dots \mid x_2 = \mathbb{E}_2[x_1] \mid x_1 = \mathbb{E}_1$

evaluation contexts:  $\mathbb{E} ::= [\cdot] \mid (\mathbb{E} e) \mid (\text{case } \mathbb{E} \text{ of } \text{alts}) \mid (\text{seq } \mathbb{E} e)$

forcing contexts:  $\mathbb{F} ::= \mathbb{E} \mid (\text{takeMVar } \mathbb{E}) \mid (\text{putMVar } \mathbb{E} e)$

# Program Equivalence

Process  $P$  is **successful** if

$$P \text{ well-formed} \wedge P \equiv \nu \vec{x}_i (x \stackrel{\text{main}}{\longleftarrow} \text{return } e \mid P')$$

**May-Convergence:** (a successful process can be reached by reduction)

Process  $P$ :  $P \Downarrow$  iff  $P$  is w.-f. and  $\exists P' : P \xrightarrow{\text{CHF},*} P' \wedge P'$  successful

Expression  $e :: \text{IO } \tau$ :  $e \Downarrow$  iff  $x \stackrel{\text{main}}{\longleftarrow} e \Downarrow$

**Should-Convergence:** (every successor is may-convergent)

Process  $P$ :  $P \Downarrow$  iff  $P$  is w.-f. and  $\forall P' : P \xrightarrow{\text{CHF},*} P' \implies P' \Downarrow$

Expression  $e :: \text{IO } \tau$ :  $e \Downarrow$  iff  $x \stackrel{\text{main}}{\longleftarrow} e \Downarrow$

## Contextual Equivalence

$$P_1 \sim_c P_2 \text{ iff } \forall \mathbb{D} : (\mathbb{D}[P_1] \Downarrow \iff \mathbb{D}[P_2] \Downarrow) \wedge (\mathbb{D}[P_1] \Downarrow \iff \mathbb{D}[P_2] \Downarrow)$$

# Simplified Expressions and Processes

## Processes

$$P, P_i \in Proc ::= P_1 \mid P_2 \mid \nu x.P \mid x \leftarrow e \mid x = e \mid x \mathbf{m} e \mid x \mathbf{m} -$$

## Expressions & Monadic Expressions

$$\begin{aligned}
 e, e_i \in Expr ::= & me \mid x \mid \lambda x.e \mid (e_1 e_2) \mid \mathbf{seq} e_1 e_2 \mid c e_1 \dots e_{\text{ar}(c)} \\
 & \mid \mathbf{case}_T e \text{ of } \dots (c_{T,i} x_1 \dots x_{\text{ar}(c_{T,i})} \rightarrow e_i) \dots \\
 & \mid \mathbf{letrec} x_1 = e_1 \dots x_n = e_n \mathbf{in} e
 \end{aligned}$$

$$\begin{aligned}
 me \in MExpr ::= & \mathbf{return} e \mid e_1 \gg= e_2 \mid \mathbf{future} e \\
 & \mid \mathbf{takeMVar} e \mid \mathbf{newMVar} e \mid \mathbf{putMVar} e_1 e_2
 \end{aligned}$$

# Simplified Expressions and Processes

## Simplified Processes

$$P, P_i \in Proc ::= P_1 \mid P_2 \mid \nu x.P \mid x \leftarrow e \mid x = e \mid x \mathbf{m} y \mid x \mathbf{m} -$$

## Simplified Expressions & Monadic Expressions

$$e, e_i \in Expr ::= me \mid x \mid \lambda x.e \mid (e_1 \mathbf{x}) \mid \mathbf{seq} e_1 \mathbf{x} \mid c \mathbf{x}_1 \dots \mathbf{x}_{\text{ar}(c)}$$

$$\mid \mathbf{case}_T e \mathbf{of} \dots (c_{T,i} \mathbf{x}_1 \dots \mathbf{x}_{\text{ar}(c_{T,i})} \rightarrow e_i) \dots$$

$$\mid \mathbf{letrec} x_1 = e_1 \dots x_n = e_n \mathbf{in} e$$

$$me \in MExpr ::= \mathbf{return} \mathbf{x} \mid \mathbf{x}_1 \gg= \mathbf{x}_2 \mid \mathbf{future} \mathbf{x}$$

$$\mid \mathbf{takeMVar} \mathbf{x} \mid \mathbf{newMVar} \mathbf{x} \mid \mathbf{putMVar} \mathbf{x}_1 \mathbf{x}_2$$



# Constructing the Abstract Machine

Modular construction in three steps:



- evaluates pure expressions
- deterministic
- treats monadic operators like data constructors
- slight modification of Sestoft's mark 1

# Constructing the Abstract Machine

Modular construction in three steps:



- adds storage (MVars)
- monadic operations are executed
- uses  $M1$  for purely functional subevaluations
- single-threaded

# Constructing the Abstract Machine

Modular construction in three steps:



- Concurrent threads (futures)
- nondeterministic
- Globally shared bindings and MVars
- uses  $IOM1$  for thread-local evaluation

# Machine $M1$

## State

$$(\mathcal{H}, e, \mathcal{S})$$

- $\mathcal{H}$  is a **heap**: a set of shared bindings  $x \mapsto e$
- $e$  is the currently evaluated **expression**
- $\mathcal{S}$  is a **stack** (holding the evaluation context)  
entries:  $\#_{\text{app}}(x)$ ,  $\#_{\text{seq}}(x)$ ,  $\#_{\text{case}}(\text{alts})$ ,  $\#_{\text{heap}}(x)$

**Start state:** For expression  $e$ :  $(\emptyset, e, [])$

**Final state:**  $(\mathcal{H}, v, [])$  where  $v = \lambda z.e$ ,  $v = (c \dots)$ ,  
or  $v$  a monadic expression

# Machine $M1$ : Transitions

## Unwinding

$$\text{(pushApp)} \quad (\mathcal{H}, (e \ x), \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e, \#_{\text{app}}(x) : \mathcal{S})$$

$$\text{(pushSeq)} \quad (\mathcal{H}, (\text{seq } e \ x), \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e, \#_{\text{seq}}(x) : \mathcal{S})$$

$$\text{(pushAlts)} \quad (\mathcal{H}, \text{case}_T e \text{ of } alts, \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e, \#_{\text{case}}(alts) : \mathcal{S})$$

$$\text{(mkBinds)} \quad (\mathcal{H}, \text{letrec } \{x_i = e_i\}_{i=1}^n \text{ in } e, \mathcal{S}) \xrightarrow{M1} (\mathcal{H} \cup \bigcup_{i=1}^n \{x_i \mapsto e_i\}, e, \mathcal{S})$$

$$\text{(enter)} \quad (\mathcal{H} \cup \{y \mapsto e\}, y, \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e, \#_{\text{heap}}(y) : \mathcal{S})$$

## Evaluation

$$\text{(takeApp)} \quad (\mathcal{H}, \lambda x.e, \#_{\text{app}}(y) : \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e[y/x], \mathcal{S})$$

$$\text{(takeSeq)} \quad (\mathcal{H}, v, \#_{\text{seq}}(y) : \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, y, \mathcal{S}), \text{ if } v = \lambda z.e \text{ or } v = c \dots$$

$$\text{(branch)} \quad (\mathcal{H}, (c \ \vec{x}_i), \#_{\text{case}}(\dots (c \ \vec{y}_i \rightarrow e) \dots)) : \mathcal{S}) \xrightarrow{M1} (\mathcal{H}, e[x_i/y_i]_{i=1}^n, \mathcal{S})$$

$$\text{(update)} \quad (\mathcal{H}, v, \#_{\text{heap}}(y) : \mathcal{S}) \xrightarrow{M1} (\mathcal{H} \cup \{y \mapsto v\}, v, \mathcal{S})$$

if  $v = \lambda z.e$ ,  $v = (c \ \dots)$ ,  $v = x$ , or  $v$  a monadic operator

# Example

$$(\emptyset, \text{letrec } x_1 = (\lambda y.y) w, x_2 = \text{takeMVar } x_1 \text{ in } ((\lambda z.z) x_2), [])$$

$$\xrightarrow{M1, \text{mkBinds}} (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, (\lambda z.z) x_2, [])$$

$$\xrightarrow{M1, \text{pushApp}} (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \lambda z.z, [\#_{\text{app}}(x_2)])$$

$$\xrightarrow{M1, \text{takeApp}} (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, x_2, [])$$

$$\xrightarrow{M1, \text{enter}} (\{x_1 \mapsto (\lambda y.y) w\}, \text{takeMVar } x_1, [\#_{\text{heap}}(x_2)])$$

$$\xrightarrow{M1, \text{update}} (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \text{takeMVar } x_1, [])$$

Machine *IOM1* $M1$ -state:  $(\mathcal{H}, e, \mathcal{S})$ 

## State

 $(\mathcal{H}, \mathcal{M}, e, \mathcal{S}, \mathcal{I})$ 

- $\mathcal{H}$  is a heap
- $e$  is the currently evaluated expression
- $\mathcal{S}$  is a stack (holding the evaluation context)
- $\mathcal{M}$  is a **set of MVars**: filled  $x \mathbf{m} y$ , empty  $x \mathbf{m} -$
- $\mathcal{I}$  is an **IO-stack** (holding the monadic context)  
entries:  $\#_{\text{take}}, \#_{\text{put}}(x), \#_{\gg=}(x)$

**Start state:** For expression  $e :: \text{IO } \tau$ :  $(\emptyset, \emptyset, e, [], [])$ **Final state:**  $(\mathcal{H}, \mathcal{M}, \text{return } x, [], [])$

# Machine *IOM1*: Transitions

## Functional Evaluation

$$(M1) \quad (\mathcal{H}, \mathcal{M}, e, \mathcal{S}, \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}', \mathcal{M}, e', \mathcal{S}', \mathcal{I})$$

if  $(\mathcal{H}, e, \mathcal{S}) \xrightarrow{M1} (\mathcal{H}', e', \mathcal{S}')$  on machine *M1*

## Monadic Unwinding

$$(pushTake) \quad (\mathcal{H}, \mathcal{M}, takeMVar \ x, [], \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M}, x, [], \#_{take} : \mathcal{I})$$

$$(pushPut) \quad (\mathcal{H}, \mathcal{M}, putMVar \ x \ y, [], \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M}, x, [], \#_{put}(y) : \mathcal{I})$$

$$(pushBind) \quad (\mathcal{H}, \mathcal{M}, x \gg= \ y, [], \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M}, x, [], \#_{\gg=}(y) : \mathcal{I})$$

## Monadic Computation

$$(newMVar) \quad (\mathcal{H}, \mathcal{M}, newMVar \ x, [], \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M} \cup \{y \ m \ x\}, return \ y, [], \mathcal{I})$$

where *y* is a fresh variable

$$(takeMVar) \quad (\mathcal{H}, \mathcal{M} \cup \{x \ m \ y\}, x, [], \#_{take} : \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M} \cup \{x \ m \ -\}, return \ y, [], \mathcal{I})$$

$$(putMVar) \quad (\mathcal{H}, \mathcal{M} \cup \{x \ m \ -\}, x, [], \#_{put}(y) : \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M} \cup \{x \ m \ y\}, return \ (), [], \mathcal{I})$$

$$(lunit) \quad (\mathcal{H}, \mathcal{M}, return \ x, [], \#_{\gg=}(y) : \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}, \mathcal{M}, (y \ x), [], \mathcal{I})$$



# Example

$$\begin{aligned}
 & (\emptyset, \{w \mathbf{m} c\}, \text{letrec } x_1 = (\lambda y.y) w, x_2 = \text{takeMVar } x_1 \text{ in } ((\lambda z.z) x_2), [], []) \\
 \xrightarrow{IOM1, M1} & (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, (\lambda z.z) x_2, [], []) \\
 \xrightarrow{IOM1, M1} & (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, \lambda z.z, [\#_{\text{app}}(x_2)], []) \\
 \xrightarrow{IOM1, M1} & (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, x_2, [], []) \\
 \xrightarrow{IOM1, M1} & (\{x_1 \mapsto (\lambda y.y) w\}, \{w \mathbf{m} c\}, \text{takeMVar } x_1, [\#_{\text{heap}}(x_2)], []) \\
 \xrightarrow{IOM1, M1} & (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, \text{takeMVar } x_1, [], []) \\
 \xrightarrow{IOM1, \text{pushTake}} & (\{x_1 \mapsto (\lambda y.y) w, x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, x_1, [], [\#_{\text{take}}]) \\
 \xrightarrow{IOM1, M1} & (\{x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, (\lambda y.y) w, [\#_{\text{heap}}(x_1)], [\#_{\text{take}}]) \\
 \xrightarrow{IOM1, M1} & (\{x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, \lambda y.y, [\#_{\text{app}}(w), \#_{\text{heap}}(x_1)], [\#_{\text{take}}]) \\
 \xrightarrow{IOM1, M1} & (\{x_2 \mapsto \text{takeMVar } x_1\}, \{w \mathbf{m} c\}, w, [\#_{\text{heap}}(x_1)], [\#_{\text{take}}]) \\
 \xrightarrow{IOM1, M1} & (\{x_2 \mapsto \text{takeMVar } x_1, x_1 \mapsto w\}, \{w \mathbf{m} c\}, w, [], [\#_{\text{take}}]) \\
 \xrightarrow{IOM1, \text{takeMVar}} & (\{x_2 \mapsto \text{takeMVar } x_1, x_1 \mapsto w\}, \{w \mathbf{m} -\}, \text{return } c, [], [])
 \end{aligned}$$

Machine *CIOM1*

*IOM1*-state:  $(\mathcal{H}, \mathcal{M}, e, \mathcal{S}, \mathcal{I})$

## State

$$(\mathcal{H}, \mathcal{M}, \mathcal{T})$$

- $\mathcal{H}$  is a heap
- $\mathcal{M}$  is a set of MVars
- $\mathcal{T}$  is a **set of threads**

**Start state** for expression  $e :: \text{IO } \tau$ :  
 $\text{Init}(e) = (\emptyset, \emptyset, \{(x, e, [], [])^{\text{main}}\})$

**Final state:** Main-thread is of the form  $(y, \text{return } x, [], [])^{\text{main}}$

## Thread

$$(x, e, \mathcal{S}, \mathcal{I})$$

- $x$  is a variable, the **name** of the future
- $e$  is the currently evaluated expression
- $\mathcal{S}$  is a stack
- $\mathcal{I}$  is an IO-stack

Main-thread:  $(x, e, \mathcal{S}, \mathcal{I})^{\text{main}}$ .

# Machine *CIOM1*: Transitions

## Thread Evaluation

(IOM1)  $(\mathcal{H}, \mathcal{M}, \mathcal{T} \cup \{(x, e, \mathcal{S}, \mathcal{I})\}) \xrightarrow{CIOM1} (\mathcal{H}', \mathcal{M}', \mathcal{T} \cup \{(x, e', \mathcal{S}', \mathcal{I}')\})$   
 if  $(\mathcal{H}, \mathcal{M}, e, \mathcal{S}, \mathcal{I}) \xrightarrow{IOM1} (\mathcal{H}', \mathcal{M}', e', \mathcal{S}', \mathcal{I}')$  on machine *IOM1*.

## Thread Creation and Finalization

(fork)  $(\mathcal{H}, \mathcal{M}, \mathcal{T} \cup \{(x, (\text{future } y), [], \mathcal{I})\})$   
 $\xrightarrow{CIOM1} (\mathcal{H}, \mathcal{M}, \mathcal{T} \cup \{(x, (\text{return } z), [], \mathcal{I}), (z, y, [], [])\})$   
 where  $z$  is a fresh variable

(unIO)  $(\mathcal{H}, \mathcal{M}, \mathcal{T} \cup \{(x, (\text{return } y), [], [])\}) \xrightarrow{CIOM1} (\mathcal{H} \cup \{x \mapsto y\}, \mathcal{M}, \mathcal{T})$   
 if thread named  $x$  is not the main-thread

# Correctness

## May- and should-convergence on $CIOM1$

State  $S$ :

- $S \downarrow_{CIOM1}$  iff  $S \xrightarrow{CIOM1,*} S' \wedge S'$  is a final state
- $S \Downarrow_{CIOM1}$  iff  $\forall S' : S \xrightarrow{CIOM1,*} S' \implies S' \downarrow_{CIOM1}$

Expression  $e :: IO \tau$

- $e \downarrow_{CIOM1}$  iff  $\text{Init}(\sigma(e)) \downarrow_{CIOM1}$
- $e \Downarrow_{CIOM1}$  iff  $\text{Init}(\sigma(e)) \Downarrow_{CIOM1}$

where  $\sigma$  translates usual expressions into simplified expressions

### Theorem

For every expression  $e :: IO \tau$ :

$$e \downarrow \iff e \downarrow_{CIOM1} \text{ and } e \Downarrow \iff e \Downarrow_{CIOM1}$$

Proof is not obvious, since transition on the machine is more restrictive than reduction in the process calculus

# Conclusion & Further Work

## Conclusion

- Sestoft's machine for lazy evaluation can be **modularly extended** to monadic I/O and concurrency
- *CIOM1* is a **correct** abstract machine for the process calculus CHF
- Correctness w.r.t. **may- and should-convergence**
- *CIOM1* is **easy to implement**, a prototype exists

## Further Work

- **Optimize** *CIOM1* by using nameless representation and avoiding substitutions. Show correctness of the optimized machine.
- Investigate how to map *CIOM1*'s threads to **parallel architectures**